Conditional Propositions, a.k.a. Implications

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The **conditional** connective, $\to$, means "implies".
$p \to q$ means "p implies q", or "if p, then q".
(See handout "Logical Connectives" for other ways of expressing implication in English.)

It is defined by the following truth table:

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

The last two rows may be a bit mysterious. In one sense, it's just a definition, and if you take it that way, you'll do fine. But *why* is the conditional defined this way?

Explanation 1

Consider the sentence "If $x < 1$ then $x < 2$." This seems obviously true by common sense.

But in propositional logic, it's not even a proposition unless we agree on the value of x.
Suppose the value of x is known, and let p be "$x < 1$" and q be "$x < 2$".
Then $p \to q$ means "If $x < 1$ then $x < 2$." *for the particular known value of* x.

If we want propositional logic to model our common sense logic, then $p \to q$ has to be true, for *all* values of x.

So for example:
"If $(0 < 1)$ then $(0 < 2)$" has to be true.
and:
"If $(3 < 1)$ then $(3 < 2)$" has to be true too.
The last example corresponds to the last line in the truth table.

Explanation 2 (for those who have studied sets)

Let A and B be sets. The definition of $A \subseteq B$, i.e. A is a subset of B, is: if $x \in A$ then $x \in B$.
For particular values of x, A and B, let p be "$x \in A$" and q be "$x \in B$".
Then the definition of $A \subseteq B$ is that $p \to q$ for all values of x.

What if A is the empty set, $\emptyset$? In set theory, by definition, $\emptyset \subseteq B$ for all B.

So "If $x \in A$ then $x \in B$" is considered to be true for all values of x, A and B, even if $A = \emptyset$ (in which case p is false, no matter what x is).
This means that $p \to q$ is considered to be true when p is false, regardless of the truth value of q.
This last sentence corresponds to the last two rows of the truth table.

Moral: In order for propositional logic to be consistent with common sense reasoning and with set theory, the truth table for the conditional has to be the way it is.